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THEORY OF NUCLEATE POOL BOILING

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Аннотация—В данной статье принято, что тепловой поток при теплопередаче в условиях кипения состоит из трех компонентов: первый—перенос тепла пузырьками пара, второй—перенос за счет молекулярной теплопроводности жидкости и третий—перенос за счет вихревой конвекции. Последний определяется по данным о барботаже. Полученное таким образом дифференциальное уравнение дает после интегрирования соотношение между локальным диаметром пузырька и температурной разностью. В результате получено, что для достаточно больших чисел Нуссельта $(Nu) \sim (Re)^{2/3}$ в допущении, что число пузырьков обратно пропорционально радиусу ядра. Если предположить, что число пузырьков на единицу площади обратно пропорционально радиусу ядра в степени m, то для больших чисел Нуссельта (Nu) можно получить соотношение

$$(Nu) \sim (Re)^{1+m/2+m}$$

Теоретические данные удовлетворительно согласуются с экспериментальными. Кроме того, анализируется первый кризис кипения, влияние нагретых поверхностей и особенности механизма пузырькового кипения.

NOMENCLATURE

a',	thermal diffusivity of the liquid;
(Ar),	Archimedes modulus;
B_1, B_2, B_3,	dimensionless parameters, equations (2.8) and (2.13);
$B_{1,0}$,	property of the liquid, equation (4.4);
c,	constant in equation (5.5);
c_p',	specific heat of the liquid at constant pressure;
C,	constant, equation (3.5);
C',	constant, equation (6.2);
C_b,	constant, equation (1.13);
C_e,	constant, equation (2.11);
C_f,	constant, equation (1.8);
C_n,	constant, equation (1.11);
C_m,	constant, equation (B.3);
C_t,	constant, equation (1.23);
C_w,	constant, equation (1.9);
C_α,	constant, equation (1.14);
C_β,	constant, equation (1.5);
D,	local diameter of a flowing bubble;
D_0,	diameter of a bubble departing from the surface;
$D^+ = D/D_0$,	dimensionless bubble diameter;
f,	frequency of bubble emission;
g,	acceleration due to gravity;
Δi,	latent heat of vaporization;
k,	exponent in equation (5.5);
K,	force of friction acting on a bubble;
(Kr),	Kruzhilin modulus;
(Ku),	Kutateladze modulus;
l,	characteristic dimension of the heat-transfer process, equation (1.4);
l_c,	characteristic dimension due to the properties of the heated surface, equation (2.25);
m,	exponent in equation (2.28);
n,	number of bubbles per unit area;
(Nu),	Nusselt modulus;
p,	pressure;
(Pr),	Prandtl modulus;
p_s'	$= (\mathrm{d}p/\mathrm{d}T)_{T=T_s}$;
q,	heat flux per unit area;
q', q'', q''',	components of the heat flux due to flow of bubbles, conduction and eddy convection, respectively;
R_a,	activation radius of a nucleus;
(Re),	Reynolds modulus for boiling heat transfer;
$(Re)_D$,	Reynolds modulus for heat transfer on flowing bubbles;

$(Re)_f$, Reynolds modulus due to the frequency of bubble emission, equation (1.6);
s, exponent in equation (1.14);
t, mean temperature of the liquid;
t_s, saturation temperature;
Δt, difference between the temperature of the surface and the saturation temperature;
T_s, absolute saturation temperature;
V, volume of a flowing bubble;
V_0, volume of a bubble at the moment of departure;
$V^+ = V/V_0$, dimensionless bubble volume;
w, relative velocity of a bubble;
w', velocity of the liquid;
w'', absolute velocity of a bubble;
X, dimensionless parameter, equation (2.19);
X_0, property of the liquid, equation (4.6);
y, co-ordinate normal to the heated surface;
Y, dimensionless parameter, equation (2.18);
Y_0, property of the liquid, equation (4.5);
α, heat-transfer coefficient for nucleate boiling;
α_D, heat-transfer coefficient for flowing bubbles;
β, contact angle;
ϵ, eddy diffusivity;
ζ, friction factor;
ϑ, temperature difference between liquid and vapour;
λ', thermal conductivity of the liquid;
ν', kinematic viscosity of the liquid;
ρ', ρ'', mass densities of the liquid and of the vapour, respectively;
σ, surface tension;
τ, time;
ϕ, constant, equation (4.7);
ψ, constant, equation (4.8);

Subscript
$kr,1$ the first crises (burnout) of boiling.

1. INTRODUCTION

THE SUBJECT of the following analysis is the heat transfer under conditions of nucleate pool boling in a superheated liquid on a horizontally submerged flat plate. Let n be the number of active sites on the heated surface. Over each active site there flows a column of bubbles (Fig. 1), each of them being characterized by its

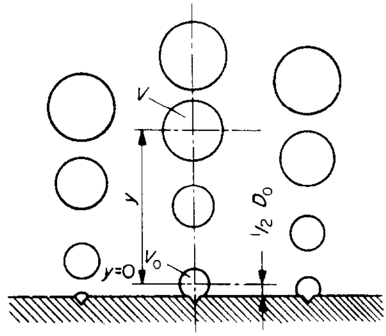

FIG. 1.

volume V, its position y, and its velocity w''. Instead of the bubble volume one may introduce the "mean" diameter

$$D = \left(\frac{6V}{\pi}\right)^{1/3}, \tag{1.1}$$

suggesting that the bubble is spherical, although in fact its shape is varied, and changes during the flow.

Let V_0 be the break off volume of the bubble. The distance

$$\frac{1}{2} D_0 = \frac{1}{2}\left(\frac{6V_0}{\pi}\right)^{1/3} \tag{1.2}$$

is used to determine the position y of the bubble centre (see Fig. 1).

According to Fritz and Bashforth [1, 2] it is

$$D_0 = C_\beta \,.\, l, \tag{1.3}$$

where

$$l = \left[\frac{\sigma}{g(\rho' - \rho'')}\right]^{1/2} \tag{1.4}$$

and

$$C_\beta \approx 0{\cdot}0209 \,.\, \beta \tag{1.5}$$

where β is the contact angle in degrees.

The frequency of bubble emission f may be evaluated from the frequency Reynolds modulus

$$(Re)_f = \frac{fD_0 l}{\nu'}. \qquad (1.6)$$

The latter is the function of the Archimedes modulus

$$(Ar) = \frac{gl^3}{\nu'^2} \cdot \frac{\rho' - \rho''}{\rho'}, \qquad (1.7)$$

namely

$$(Re)_f = C_f \,.\, (Ar)^{1/2}. \qquad (1.8)$$

According to Zuber [3] it is $C_f = 0{\cdot}59$, and according to McFadden and Grassmann [4] $C_f = 0{\cdot}56\ \sqrt{(C_\beta)}$.

The bubble velocity w'' may be evaluated as follows. Let w be the relative velocity of the bubble in liquid. According to Peebles and Garber [5] it is

$$w = C_w \left[\frac{gl\,(\rho' - \rho'')}{\rho'}\right]^{1/2}, \quad C_w = 1{\cdot}18. \qquad (1.9)$$

The absolute velocity w'' is somewhat smaller because of the interference of neighbouring active sites. Suppose at first that the bubbles break off simultaneously; then each column flows in a channel of area $1/n$. In real conditions this area must be larger, namely C_n/n, where $C_n > 1$ is a correction factor.

During the motion of the bubble in time interval $\mathrm{d}\tau$ a displacement $\mathrm{d}y = w''\,\mathrm{d}\tau$ causes the release of space

$$\frac{\pi}{4} D^2\,\mathrm{d}y,$$

into which there enters the liquid, flowing through the free (not occupied by the vapour) sector of the channel. The area of this sector is

$$\frac{C_n}{n} - \frac{\pi}{4} D^2.$$

Therefore

$$w'\,\mathrm{d}\tau \left(\frac{C_n}{n} - \frac{\pi}{4} D^2\right) + \frac{\pi}{4} D^2\,\mathrm{d}y = 0,$$

where w' is the velocity of the liquid. Thus

$$w' = -w'' \cdot \frac{(\pi/4)\,D^2}{(C_n/n) - (\pi/4)\,D^2}, \quad w'' = \frac{\mathrm{d}y}{\mathrm{d}\tau}, \qquad (1.10)$$

and

$$w = w'' - w' = \frac{w''}{1 - (\pi n/4C_n)\,.\,D^2}$$

or

$$w'' = w\left(1 - \frac{\pi n}{4C_n}\,.\,D^2\right). \qquad (1.11)$$

The growth of the bubble volume V is governed by heat transfer in liquid. According to Bošnjaković [6] it is

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = \frac{2\,\alpha_D\,.\,\vartheta}{\Delta i\,.\,\rho''}, \qquad (1.12)$$

where ϑ is the mean temperature difference between the liquid and the vapour, and α_D is the heat-transfer coefficient. The former will be assumed as the temperature difference at the place y. This assumption is substantiated for smaller temperature gradients $\mathrm{d}t/\mathrm{d}y$ in liquid. In the vicinity of the heated surface ($y = 0$) however, i.e. for the bubble in break-off position, the temperature difference is supposed to be greater because of the abrupt temperature profile at the heated surface. We will assume

$$\vartheta_{y=0} = C_b\,.\,\Delta t, \qquad (1.13)$$

where Δt is the effective temperature difference between the heated surface and the vapour.

The heat-transfer coefficient α_D will be evaluated from

$$\frac{\alpha_D\,.\,D}{\lambda'} = C_\alpha\,.\,(Pr)^{1/3}\,.\,(Re)_D^s, \quad (Pr) = \frac{\nu'}{a'}, \qquad (1.14)$$

where

$$(Re)_D = \frac{wD}{\nu'} = C_w\,.\,\frac{D}{l}\,.\,(Ar)^{1/2} \qquad (1.15)$$

according to (1.9).

The heat flux q consists of three components. The first,

$$q' = V\,\Delta i\,.\,\rho''\,.\,nf, \qquad (1.16)$$

results from the motion of bubble columns. This component is the main part of the heat flux at some distance from the heated surface. At the limit $y = 0$, however, its value is small.

The second component,

$$q'' = -\,\lambda'\,\frac{\mathrm{d}t}{\mathrm{d}y}, \qquad (1.17)$$

is due to the molecular heat conduction in liquid. This portion is of no importance for $y > 0$. In the vicinity of the surface, however, this component plays a significant role; it governs, in certain sense, the value of the constant C_b in equation (1.13). In the following analysis, made for $y > 0$, the component q'' will be neglected.

The last component, q''', is due to the turbulent convection. It is given by

$$q''' = - \epsilon c_p' \rho' \frac{\mathrm{d}t}{\mathrm{d}y}, \qquad (1.18)$$

where ϵ is the eddy diffusivity. The latter may be put proportional to the product of velocity w and dimension D

$$\epsilon \sim wD,$$

according to the concepts of turbulence [7]. A similar assumption might be made;

$$\epsilon \sim \frac{\mathrm{d}I}{\mathrm{d}M} \cdot D \qquad (1.19)$$

where $\mathrm{d}I$ is the impulse of friction force

$$K = \frac{\zeta}{8} \cdot \rho' w^2 \pi D^2 \qquad (1.20)$$

acting on a single bubble in time interval $\mathrm{d}\tau$:

$$\mathrm{d}I = K \,\mathrm{d}\tau.$$

The friction factor is equal

$$\zeta = 0{\cdot}967 \frac{D}{l} \qquad (1.21)$$

according to Peebles and Garber [5]. Furthermore $\mathrm{d}M$ is the mass of swirling liquid, which is equal to

$$\mathrm{d}M = \frac{w\rho'}{n} \mathrm{d}\tau.$$

Substitution into (1.19) yields

$$\epsilon \sim \zeta\, wn\, D^3$$

or

$$\epsilon = C_t' \,.\, \zeta\, wn\, D^3.$$

Using equations (1.9) and (1.21) we obtain

$$\epsilon = C_t \,.\, \nu' n \,.\, \frac{D^4}{l^2} (Ar)^{1/2}, \qquad (1.22)$$

where

$$C_t = 0{\cdot}967 \,.\, C_t' \, C_w. \qquad (1.23)$$

Substitution of equation (1.22) into equation (1.18) yields

$$q''' = - \lambda' \,.\, \frac{\mathrm{d}t}{\mathrm{d}y} \,.\, C_t\, n \frac{D^4}{l^2} (Pr)\,(Ar)^{1/2}. \qquad (1.24)$$

The described model of nucleate pool boiling formed the subject of previous investigations of the author [8], based on the preceding assumptions. The theory [8] gives quite correct results and will be developed and improved in this paper. In comparison with the analysis in [8], this theory will neglect the influence of hydrostatic pressure on the saturation temperature t_s. This influence, as it was found [8], is of secondary importance in problems of nucleate pool boiling. It will be assumed therefore

$$\vartheta = t - t_s, \quad t_s = \text{const}, \qquad (1.25)$$

where t is the temperature of superheated liquid.

2. INTEGRATION OF THE HEAT FLUX EQUATION

The total heat flux is obtained by summation of equations (1.16) and (1.24). Using equation (1.1) we get

$$q = \frac{\pi}{6} D^3 \,\Delta i \,.\, \rho'' nf - \lambda' \frac{\mathrm{d}t}{\mathrm{d}y} C_t \,.\, n \frac{D^4}{l^2} (Pr)\,(Ar)^{1/2} = \text{const}. \qquad (2.1)$$

From equations (1.12), (1.14) and (1.15) it follows

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = 2\vartheta \,.\, \frac{\lambda'}{\Delta i \,.\, \rho'' \,.\, l} \,.\, C_\alpha C_w^s \,.\, \left(\frac{D}{l}\right)^{s-1} \,.\, (Pr)^{1/3} (Ar)^{s/2}. \qquad (2.2)$$

But we have

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = \frac{\mathrm{d}D}{\mathrm{d}y} \,.\, \frac{\mathrm{d}y}{\mathrm{d}\tau} = w'' \frac{\mathrm{d}D}{\mathrm{d}y}$$

or

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = C_w \,.\, \frac{\nu'}{l} (Ar)^{1/2} \,.\, \left(1 - \frac{\pi n}{4C_n} D^2\right) \,.\, \frac{\mathrm{d}D}{\mathrm{d}y} \qquad (2.3)$$

by virtue of equations (1.9) and (1.11). Now, since

$$\frac{dt}{dy} = \frac{d\vartheta}{dy} = \frac{d\vartheta}{dD} \cdot \frac{dD}{dy},$$

we obtain by substitution of equation (2.2) into equation (2.3)

$$\frac{dt}{dy} = 2\vartheta \cdot \frac{d\vartheta}{dD} \cdot C_\alpha\, C_w^{s-1} \cdot (Pr)^{-2/3} \cdot (Ar)^{s-1/2} \cdot \frac{(c_p'\rho'/\Delta i \cdot \rho'')}{1-(\pi n/4C_n)\, D^2} \cdot \left(\frac{D}{l}\right)^{s-1} \tag{2.4}$$

with

$$\frac{dD}{dy} = 2\vartheta \cdot C_\alpha\, C_w^{s-1} \cdot (Pr)^{-2/3} \cdot (Ar)^{s-1/2} \cdot \frac{(c_p'\rho'/\Delta i \cdot \rho'')}{1-(\pi n/4C_n)\, D^2} \cdot \left(\frac{D}{l}\right)^{s-1}. \tag{2.5}$$

Substitution of equation (2.4) into equation (2.1) yields

$$B_1 = D^{+^s} - B_2' \cdot \frac{D^{+3+s}}{1 - B_3\, D^{+2}} \cdot \frac{d\vartheta^2}{dD^+}, \tag{2.6}$$

where

$$D^+ = \frac{D}{D_0} = \frac{D}{C_\beta \cdot l} \tag{2.7}$$

is the dimensionless bubble diameter, and

$$\left.\begin{aligned} B_1 &= \frac{6q}{\pi\, D_0^2\, \Delta i \cdot \rho''\, nf}, \\ B_2' &= \frac{6}{\pi} C_\alpha\, C_t\, C_\beta^s\, C_w^{s-1} \cdot (Pr)^{-2/3}\, (Ar)^{s/2} \cdot \frac{\nu'}{f\, D_0\, l} \cdot \left(\frac{c_p'\rho'}{\Delta i \cdot \rho''}\right)^2, \\ B_3 &= \frac{\pi\, n\, D_0^2}{4C_n}. \end{aligned}\right\} \tag{2.8}$$

Equation (2.6) can be easily integrated with the initial condition (1.13), that is

$$D^+ = 1 \quad \text{for} \quad \vartheta^2 = C_b^2\, (\Delta t)^2. \tag{2.9}$$

We obtain

$$B_2'\, [C_b^2\, (\Delta t)^2 - \vartheta^2] = -\frac{1}{1-s}(D^{+1-s} - 1) - \frac{B_1}{2+s}(D^{+-(2+s)} - 1) + B_3 \left[\frac{1}{3-s}(D^{+3-s} - 1) + \frac{B_1}{s}(D^{+-s} - 1)\right] \tag{2.10}$$

for $0 < s < 1$. Solutions for $s = 0$ and $s = 1$ are given in Appendix A.

On the liquid–vapour boundary it should be $d\vartheta/dy = 0$, or $d\vartheta^2/dD^+ = 0$, wherefore $q''' = 0$ and

$$q = q' = \frac{\pi}{6} D_{\max}^3 \cdot \Delta i \cdot \rho''\, nf,$$

whence

$$\left(\frac{D_{\max}}{D_0}\right)^3 = D_{\max}^{+^s} = B_1.$$

The temperature difference on the liquid–vapour boundary will be denoted

$$\vartheta_e = C_b\, C_e\, \Delta t. \tag{2.11}$$

Substituting $\vartheta = \vartheta_e$, $D^+ = B_1^{1/3}$ into equation (2.10) yields

$$B_2 = B_1 - \frac{3}{1-s} \cdot B_1^{1-s/3} + \frac{2+s}{1-s} - \frac{2+s}{s} \cdot B_3 \cdot \left[B_1 - \frac{3}{3-s} \cdot B_1^{3-s/3} + \frac{s}{3-s}\right], \tag{2.12}$$

where

$$B_2 = (2+s)\, B_2' \cdot C_b^2\, (1 - C_e^2)\, (\Delta t)^2 = \frac{6\,(2+s)}{\pi C_f} C_\alpha\, C_t\, C_\beta^s\, C_w^{s-1}\, C_b^2\, (1 - C_e^2) \cdot (Pr)^{-2/3}\, (Ar)^{s-1/2} \cdot \left(\frac{c_p'\, \rho'\, \Delta t}{\Delta i \cdot \rho''}\right)^2. \tag{2.13}$$

In this formula the equations (1.6) and (1.8) are taken into account.

We introduce now the dimensionless groups

$$(Nu) = \frac{\alpha l}{\lambda'} = \frac{ql}{\lambda'\, \Delta t}, \tag{2.14}$$

$$(Re) = \frac{ql}{\Delta i \cdot \rho''\nu''}, \tag{2.15}$$

$$(Kr) = \frac{c_p'\, \rho'\, \Delta t}{2\, \Delta i \cdot \rho''\, l\, l_c\, n}. \tag{2.16}$$

The group (Kr), in which l_c denotes the characteristic dimension, representing the microgeometry of the heated surface, is called—according to a recent proposal—the Kruzhilin modulus.

The groups B_1, $B_2 = Y^2$ and $B_3 = 2XY$ may now be expressed in terms of (Nu), (Re), (Kr), (Pr), (Ar), namely

$$B_1 = \frac{12}{\pi C_f C_\beta^2}\cdot\frac{l_c}{l}\cdot\frac{(Nu)(Kr)}{(Pr)(Ar)^{1/2}}, \tag{2.17}$$

$$Y = \sqrt{B_2} = \sqrt{\left[\frac{6(2+s)}{\pi C_f} C_\alpha C_t C_\beta^s C_w^{s-1} C_b^2 (1 - C_e^2)\right]}\, (Pr)^{2/3} (Ar)^{s-1/4}\cdot\frac{(Re)}{(Nu)}, \tag{2.18}$$

$$X = \frac{B_3}{2Y} = \frac{\pi C_\beta^2}{16 C_n}\cdot\sqrt{\left[\frac{\pi C_f}{6(2+s) C_\alpha C_t C_\beta^s C_w^{s-1} C_2^b (1 - C_e^2)}\right]}\cdot\frac{(Pr)^{1/3}(Ar)^{1-s/4}}{(Kr)}\cdot\frac{l}{l_c}. \tag{2.19}$$

Substituting X and Y into equation (2.12) yields

$$Y^2 + 2Y\cdot\frac{2+s}{s}\cdot X\cdot\left[B_1 - \frac{3}{3-s}\cdot B_1^{3-s/3} + \frac{s}{3-s}\right] - \left[B_1 - \frac{3}{1-s}\cdot B^{1-s/3} + \frac{2+s}{1-s}\right] = 0, \tag{2.20}$$

whence

$$Y = \sqrt{\left[B_1 - \frac{3}{1-s} B_1^{1-s/3} + \frac{2+s}{1-s} + X^2\cdot\left(\frac{2+s}{s}\right)^2\cdot\left(B_1 - \frac{3}{3-s}\cdot B_1^{3-s/3} + \frac{s}{3-s}\right)^2\right]} - X\cdot\frac{2+s}{s}\cdot\left[B_1 - \frac{3}{3-s}\cdot B_1^{3-s/3} + \frac{s}{3-s}\right]. \tag{2.21}$$

This solution is valid only for

$$1 - B_3 D^{+2} > 0.$$

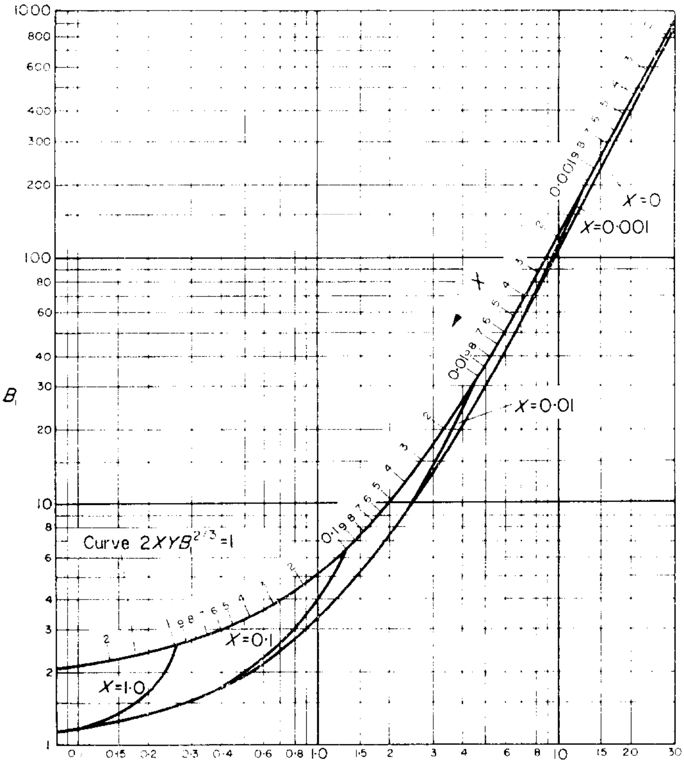

FIG. 2.

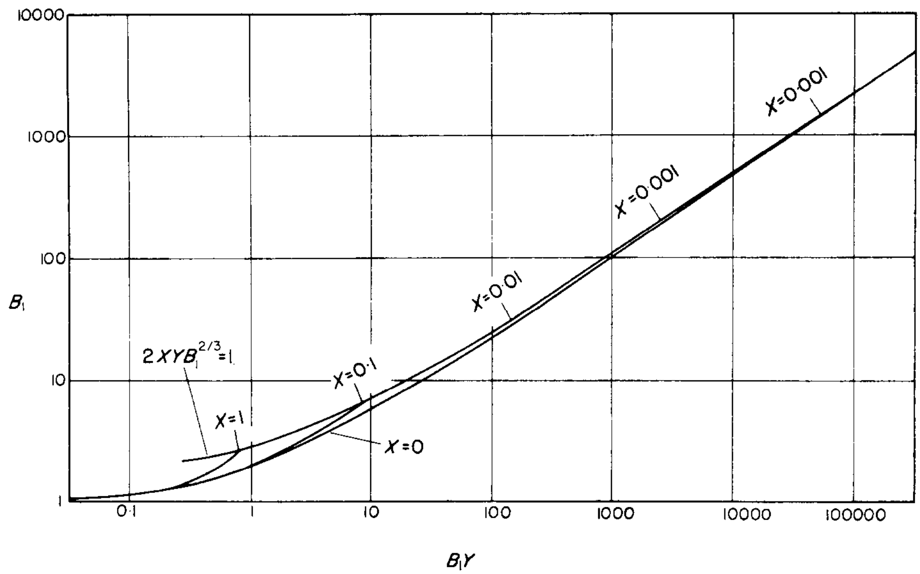

FIG. 3.

Taking into account equation (2.11) we obtain the critical condition

$$1 - B_3\, B_1^{2/3} = 0$$

or

$$2\, XY\, B_1^{2/3} = 1. \tag{2.22}$$

The relationships $B_1(Y)$ and $B_1(B_1 Y)$ are shown in Figs. 2 and 3 for $s = 1/2$. Figure 4 contains a comparison of relations $B_1(Y)$ for $X = 0$ and $s = 0$; $1/2$; 1. It is seen that the influence of exponent s is small.

If $B_1 > 10$ the solution (2.21) yields approximately

$$Y \approx \sqrt{B_1} \tag{2.23}$$

or

$$\frac{(Nu)}{(Re)^{2/3}} = \left[\frac{2+s}{2} C_\alpha\, C_t\, C_\beta^{2+s}\, C_b^2\, (1 - C_e^2)\, C_w^{s-1}\right]^{1/3} \cdot \left(\frac{l}{l_c}\right)^{1/3} (Pr)^{7/9}\, (Ar)^{s/6}\, (Kr)^{-1/3}. \tag{2.24}$$

The relation $(Nu) \sim (Re)^{2/3}$ was observed by many investigators (e.g. [9, 10]), which indicates that the Kruzhilin modulus (Kr) must be independent of the temperature difference Δt as well as independent of the heat flux q, assuming

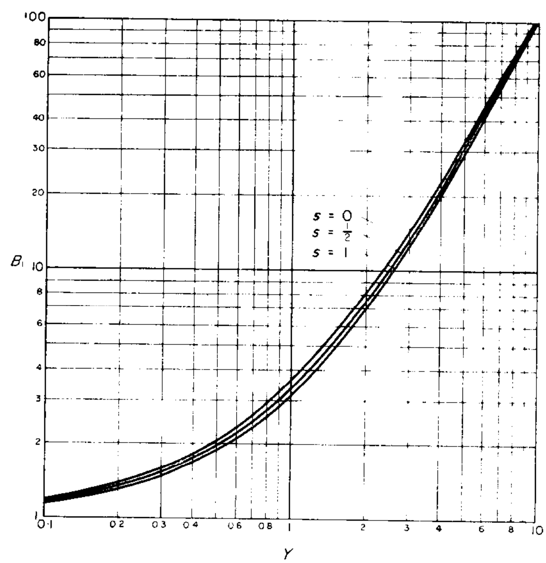

FIG. 4.

that $C_b = \text{const}$. This condition may be easily fulfilled if we assume the reciprocal proportionality between the bubble population n and the activation radius of a nucleus R_a,

$$n = \frac{1}{l_c\, R_a}, \tag{2.25}$$

where l_c is the characteristic dimension, due to the properties of the heated surface already mentioned.

The activation radius is equal to

$$R_a = \frac{2\sigma T_s}{\Delta i \, . \, \rho'' \, \Delta t}. \tag{2.26}$$

Substitution of equations (2.25) and (2.26) into equation (2.16) yields finally

$$(Kr) = (Ku), \quad (Ku) = \frac{c_p' \rho'^2 g l}{p_s' \Delta i \, . \, \rho''}, \tag{2.27}$$

where equation (1.4) and the Clausius–Clapeyron equation

$$p_s' = \left(\frac{\mathrm{d}p}{\mathrm{d}T}\right)_{T=T_s} = \frac{\Delta i \, . \, \rho'' \rho'}{T_s(\rho' - \rho'')}$$

were taken into account. The dimensionless group (Ku) is called—according to a recent proposal of the author—the Kutateladze modulus.

Some investigators [11–14] observed, however, that the bubble population grows more quickly with the growth of temperature difference. To analyse that problem we assume

$$n = \frac{1}{l_c^{2-m} R_a^m}. \tag{2.28}$$

By substitution in equation (2.16) we obtain

$$(Kr) = (Ku)^m \, . \left[2 \frac{l}{l_c} \cdot \frac{(Nu)}{(Re)(Pr)}\right]^{m-1}, \tag{2.29}$$

which substituted again in equation (2.24) yields

$$\frac{(Nu)}{(Re)^{1+m/2+m}} = \left[\frac{2+s}{2} C_\alpha C_t C_\beta^{2+s} C_w^{s-1} C_b^2 (1 - C_e^2)\right]^{1/2+m} \cdot \left(\frac{l}{l_c}\right)^{2-m/2+m} . (Pr)^{4+3m/6+3m} . (Ku)^{-(m/2+m)}. \tag{2.30}$$

For $m = 3$ we obtain $(Nu) \sim (Re)^{0 \cdot 8}$.

3. THE FIRST CRISIS OF BOILING

It has already been pointed out that the obtained solution is valid for

$$1 - B_3 B_1^{2/3} > 0$$

only. The criterion

$$1 - B_3 B_1^{2/3} = 0, \quad \text{or} \quad 2\,XYB_1^{2/3} = 1 \tag{3.1}$$

is identical with the condition $w'' = 0$ for $D^+ = B_1^{1/3}$.

Thus, if the vapour flow is stopped, the liquid will quickly evaporate forming a vapour layer near the heated surface. Although there exist other criteria of the burnout [15], we may regard the condition (3.1) as a criterion of the first crisis of boiling.

In the region of $B_1 > 10$ the relation (2.23) may be used. Substitution of equation (2.23) into equation (3.1) yields

$$B_{1Kr,1} = \left(\frac{1}{2X}\right)^{6/7} \tag{3.2}$$

and

$$(YB_1)_{Kr,1} = \left(\frac{1}{2X}\right)^{9/7} \tag{3.3}$$

or

$$(Re)_{Kr,1} = C \, . \left(\frac{l_c}{l}\right)^{2/7} (Kr)^{2/7} . (Pr)^{-(2/21)} . (Ar)^{4+s/14}, \tag{3.4}$$

with

$$C = \frac{\pi C_f C_\beta^2}{12} \cdot \left(\frac{8 C_n}{\pi C_\beta^2}\right)^{2/7} . \left[\frac{6(2+s)}{\pi C_f} C_\alpha C_t C_\beta^s C_b^2 (1 - C_e^2) C_w^{s-1}\right]^{1/7}. \tag{3.5}$$

From equations (2.29) and (2.30) it follows

$$(Kr) \sim (Re)^{1-m/2+m},$$

which substituted in equation (3.4) yields

$$(Re)_{Kr,1} \sim (Ar)^{(14+7m/12+9m) \, . \, (4+s/14)}. \tag{3.6}$$

Assuming $m = 1$, $s = 1/2$ we get

$$(Re)_{Kr,1} \sim (Ar)^{13/28} \approx (Ar)^{1/2},$$

which is confirmed experimentally [16]; it agrees also with other theories [15]. Labuntzov [17] has suggested

$$(Re)_{Kr,1} \sim (Ar)^{4/9};$$

this corresponds to the value $m = 14/9 \approx 1 \cdot 56$ at $s \approx 1/2$ assumed.

4. COMPARISON WITH EXPERIMENTAL DATA FOR WATER

The comparison was done for $m = 1$ and $s = 1/2$. It was assumed $C_f = 0{\cdot}59$ (after Zuber [3]), $C_w = 1{\cdot}18$ and $C_n = 1{\cdot}258$.

The parameters B_1, Y and X may be expressed as follows

$$B_1 = \phi \, B_{1,0} \, (Nu), \tag{4.1}$$

$$Y = \psi \, Y_0 \frac{(Re)}{(Nu)}, \tag{4.2}$$

$$X = \frac{X_0}{\phi \, \psi}, \tag{4.3}$$

where $B_{1,0}$, Y_0 and X_0 are factors, which depend only upon the properties of the boiling medium; namely

$$B_{1,0} = \frac{12}{\pi \, C_f \, l} \cdot \frac{(Ku)}{(Pr)\,(Ar)^{1/2}}, \tag{4.4}$$

$$Y_0 = \sqrt{\left[\frac{15}{\pi \, C_f \, C_w^{1/2}}\right]} \cdot (Pr)^{2/3} \cdot (Ar)^{-1/8}, \tag{4.5}$$

$$X_0 = \frac{\pi}{16 \, C_n} \cdot \sqrt{\left[\frac{\pi \, C_f \, C_w^{1/2}}{15}\right]} \cdot \frac{(Pr)^{1/3}\,(Ar)^{1/8}}{(Ku)} \cdot l. \tag{4.6}$$

The graph $B_{1,0}$; Y_0; X_0; $B_{1,0} \, . \, Y_0$ vs. pressure p is given in Fig. 5 for water.

The next group of factors

$$\phi = \frac{l_c}{C_\beta^2}, \tag{4.7}$$

$$\psi = \sqrt{[C_\alpha \, C_t \, C_b^2 \, C_\beta^{1/2} \, (1 - C_e^2)]} \tag{4.8}$$

contains factors, which depend upon the roughness and material of the surface (l_c) and upon the conditions of the contact (C_β). The numbers C_t, C_α, C_b and C_e not known at the outset, are included. It is to be emphasized that the separate values of these numbers are of secondary interest.

Comparison with the experimental data should be done in this way. Suitable values ϕ and ψ must be found to make theory agree with experiment. From the latter we know the values of

$$B_{1,0} \, . \, (Nu) = \frac{B_1}{\phi},$$

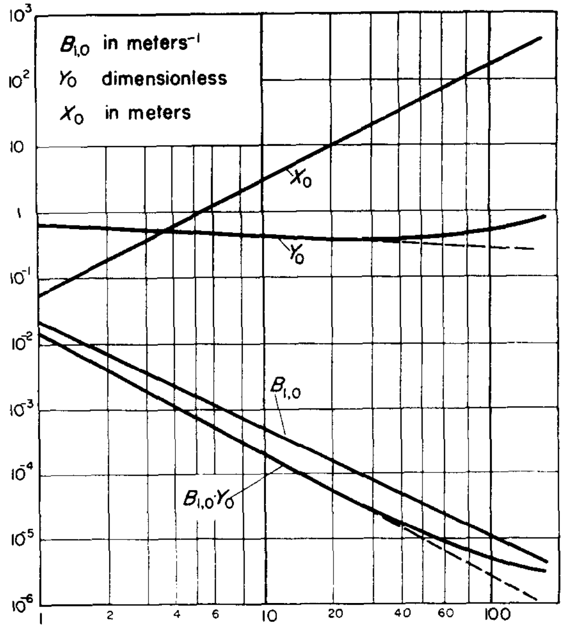

FIG. 5.

$$Y_0 \, . \, \frac{(Re)}{(Nu)} = \frac{Y}{\psi},$$

and

$$X_0 = X \, \phi \, \psi.$$

Now the graph

$$\log \frac{B_1}{\phi} = f\left(\log \frac{Y}{\psi}\right)$$

should be drawn on tracing paper. By superposing this tracing paper on the graph from Fig. 2 and shifting it in the direction of the co-ordinates a position may be found, in which the experimental points are possibly near to the theoretical curve. The logarithmic scales of the graphs must both be the same, of course. Now, from the mutual displacements of the co-ordinate systems the coefficients ϕ and ψ may be evaluated, since

$$\log \frac{B_1}{\phi} = \log B_1 - \log \phi$$

and

$$\log \frac{Y}{\psi} = \log Y - \log \psi.$$

The values of ϕ and ψ so obtained are of course approximate. The correct values are to be calculated together with the parameter X.

The results of the comparison with the experments [18–22] are shown in Fig. 6. Figure 7

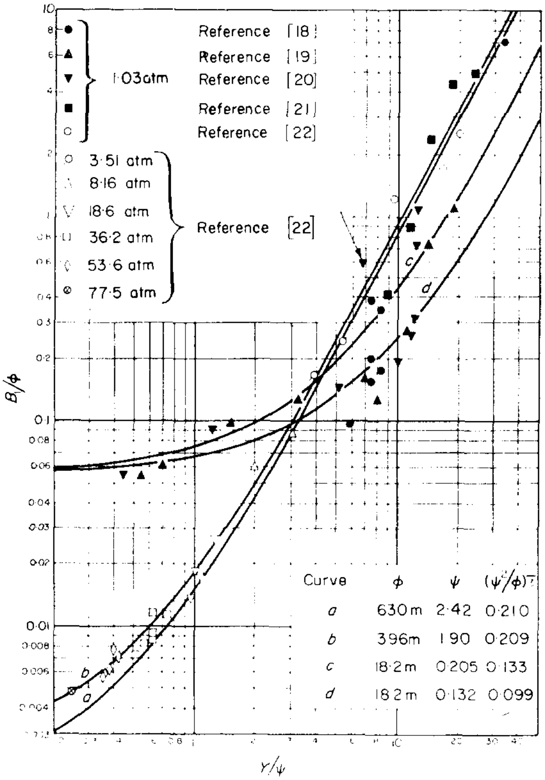

FIG. 6.

shows the relationship $(Nu) = f[(Re)]$ obtained from this theory with $\phi = 396$ m, $\psi = 1{\cdot}9$, compared with experimental data of Cichelli and Bonilla [22] for heat transfer in boiling water under pressure. This graph is reproduced from author's paper [8] and shows the influence of the depth of water layer H; the extreme values lie at $H = 0$ and at $H = 0{\cdot}5$ m. It is seen, as was indicated before, that the influence of hydrostatic pressure is small.

The functions $B_{1,0}$ and X_0 may be expressed for water by the following quite accurate formulae

$$\left.\begin{aligned} B_{1,0} &= 2{\cdot}286 \times 10^{-2} \,.\, p^{-1{\cdot}681}, \\ X_0 &= 5{\cdot}260 \times 10^{-2} p^{1{\cdot}729},\ [p] = \text{atm}, \end{aligned}\right\} \quad (4.9)$$

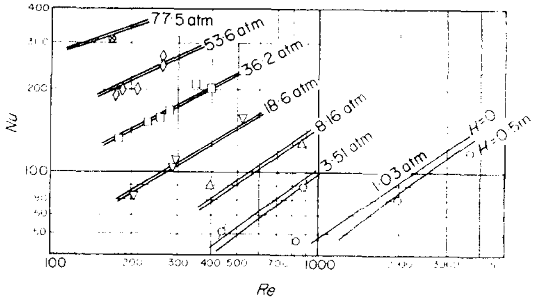

FIG. 7.

which are valid for $1 \leqslant p < 170$ atm.

An analogous expression

$$Y_0 = 0{\cdot}656 \times p^{-0{\cdot}188} \quad (4.10)$$

is valid for $1 \leqslant p < 30$ atm.

If $B_1 > 10$, then $Y \approx \sqrt{B_1}$ and

$$\psi\, Y_0 \frac{(Re)}{(Nu)} \approx [\phi\, B_{1,0}\, (Nu)]^{1/2}$$

or

$$\frac{(Nu)}{(Re)^{2/3}} \approx \left[\frac{\psi^2}{\phi} \cdot \frac{Y_0}{B_{1,0}}\right]^{1/3}. \quad (4.11)$$

Using equations (4.9) and (4.10) we obtain

$$\frac{(Nu)}{(Re)^{2/3}} \approx \left(\frac{\psi^2}{\phi}\right)^{1/3} 2{\cdot}66\, p^{0{\cdot}435}. \quad (4.12)$$

From Fig. 6 it is seen that the value of $(\psi^2/\phi)^{1/3}$ is little affected by the choice of ϕ and ψ. Assuming $(\psi^2/\phi)^{1/3} = 0{\cdot}209$ we obtain from experiments of Cichelli and Bonilla

$$\frac{(Nu)}{(Re)^{2/3}} \approx 0{\cdot}556\, p^{0{\cdot}435} \quad (4.13)$$

for water at $p < 30$ atm.

The critical heat flux may be evaluated from equation (3.3) with use of equations (4.1), (4.2) and (4.3); the result is

$$(Re)_{Kr,1} = (\phi\, \psi)^{2/7} \frac{1}{(2\, X_0)^{9/7} \,.\, B_{1,0}\, Y_0}. \quad (4.14)$$

Substitution of equations (4.9) and (4.10) yields

$$(Re)_{Kr,1} \approx (\phi\, \psi)^{2/7}\, 1200\, p^{-0{\cdot}354}. \quad (4.15)$$

Assuming $\phi = 396$ m, $\psi = 1{\cdot}9$ we obtain

$$(Re)_{Kr,1} \approx 7960\, p^{-0{\cdot}354} \quad (4.16)$$

for water at $p < 30$ atm.

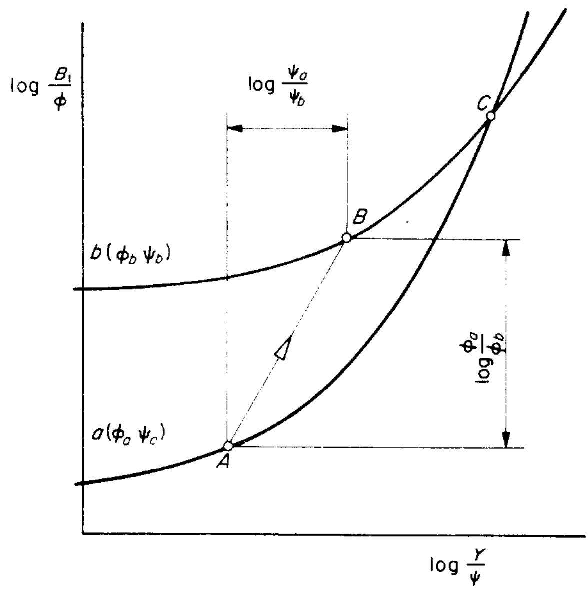

FIG. 8.

5. REMARKS ON THE INFLUENCE OF THE HEATED SURFACE

Suppose there exists a relationship $\log(B_1/\phi) = f[\log(Y/\psi)]$ for a certain fluid with $\phi = \phi_a$, $\psi = \psi_a$ ($X \approx 0$ assumed), as shown in Fig. 8 by curve a; if the roughness or material of the surface are changed we should have $\phi = \phi_b$, $\psi = \psi_b$. The corresponding curve b may be drawn by shifting of the curve a in the direction of $\log(B_1/\phi)$ by $\log(\phi_a/\phi_b)$, and in the direction of $\log(Y/\psi)$ by $\log(\psi_a/\psi_b)$ as shown in Fig. 8. The points A and B have the same values of B_1 and Y.

Now, if in both cases the contact angles, β, and consequently the values C_β are the same, according to equation (4.8) we should have $\psi_a = \psi_b$, and the displacement of the curve a into position b is vertical only. In this case the only difference may be the roughness of the surface, which influences the characteristic dimension l_c. Thence

$$\log\frac{\phi_a}{\phi_b} = \log\frac{l_{c,a}}{l_{c,b}}, \quad \log\frac{\psi_a}{\psi_b} = 0. \tag{5.1}$$

Suppose now that the surfaces have the same characteristic dimension l_c. Then, according to equation (4.7), (4.8) and (1.5)

$$\log\frac{\phi_a}{\phi_b} = 2\log\frac{C_{\beta,b}}{C_{\beta,a}} = 2\log\frac{\beta_b}{\beta_a} \tag{5.2}$$

and

$$\log\frac{\psi_a}{\psi_b} = \frac{1}{4}\log\frac{C_{\beta,a}}{C_{\beta,b}} = -\frac{1}{4}\log\frac{\beta_b}{\beta_a}. \tag{5.3}$$

The slope of the shifting straight line AB, shown in Fig. 9, is $-1/8$.

Similar relations exist in the diagram

$$\log\frac{B_1}{\phi} = f\left[\log\frac{Y B_1}{\phi\,\psi}\right],$$

which corresponds to the relation $(Nu) = f[(Re)]$. This is illustrated in Fig. 10. For $l_{c,a} = l_{c,b}$ we obtain

$$\log\frac{\phi_a\,\psi_a}{\phi_b\,\psi_b} = \frac{7}{4}\log\frac{\beta_b}{\beta_a}, \tag{5.4}$$

so that the slope of the shifting line AB is 7/8.

In general, the curves a and b may intersect (point C in Fig. 8). It means that in point C the same values of (Nu) and (Re) exist for both cases. The slopes of the tangents are different, however.

Although the relationships $\log B_1 = f(\log Y)$ are not represented by a straight line it is possible to write down the equation of the tangent for every point of the curve. In the vicinity of that point the tangent is an approximation of the analysed relationship. We thus obtain an equation

$$B_1 = c\,.\,(B_1\,Y)^k, \tag{5.5}$$

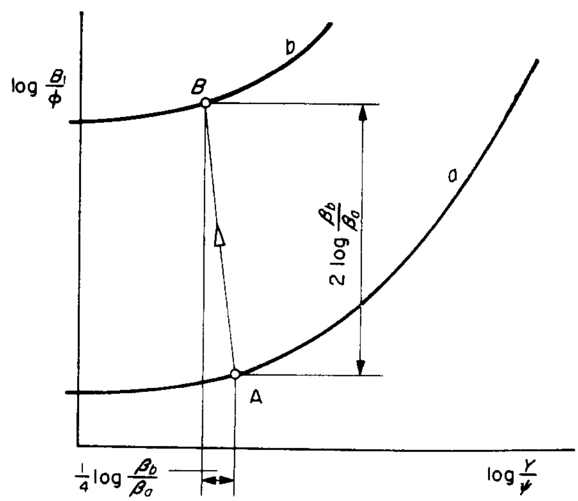

FIG. 9.

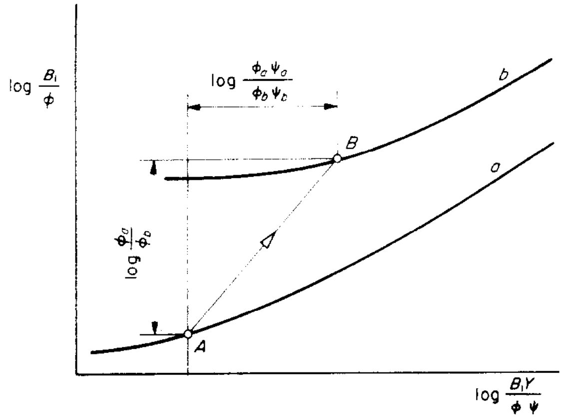

FIG. 10.

where

$$k = \frac{d \log B_1}{d \log (B_1 Y)}, \quad c = \frac{B_1}{(B_1 Y)^k}. \tag{5.6}$$

The functions k and c may be obtained from equation (2.21); for $X \approx 0$ and $s = 1/2$ it is

$$k = \left[1 + \frac{B_1 - B_1^{1/6}}{2Y^2}\right]^{-1}. \tag{5.7}$$

Figure 11 shows the relationship c and k vs. $B_1 Y$.

If $m = 1$ the equation (5.5) corresponds to the usual equation

$$(Nu) = c' . (Re)^k. \tag{5.8}$$

If experiments are done within an interval of the heat flux with the same liquid at the same conditions save the material of heated surface, the curves $\log (B_1/\phi) = f[\log (B_1 Y/\phi\, \psi)]$ lie in the same interval of $(B_1 Y/\phi\, \psi)$. The curves for material with greater values of the contact angle β are shifted up and to the right (curve b in Fig. 10). In the examined interval of $(B_1 Y/\phi\, \psi)$ the upper curves have smaller slopes. This is confirmed by recent experiments of Kostin [23]. This investigator has found for nucleate boiling of water on aluminium surface

$$a = 154{\cdot}9\, q^{0{\cdot}369},$$

and on nickel surface

$$a = 4{\cdot}04\, q^{0{\cdot}675}$$

at $q = (2 \ldots . 15)\ 10^4$ kcal/m²h. The roughness of the surfaces was the same.

Now, it was observed by the author that the contact angle of water on aluminium is considerably greater than on copper, brass or nickel. This fact should be stressed since there are no data on this topic in literature.

6. THE MECHANISM OF NUCLEATE BOILING AS RESULT OF THE MODEL ASSUMED

For technical purposes it is sufficient to know the relationship $(Nu) = f[(Re)]$, which was obtained and discussed above. However, to clarify the physical problem of nucleate pool

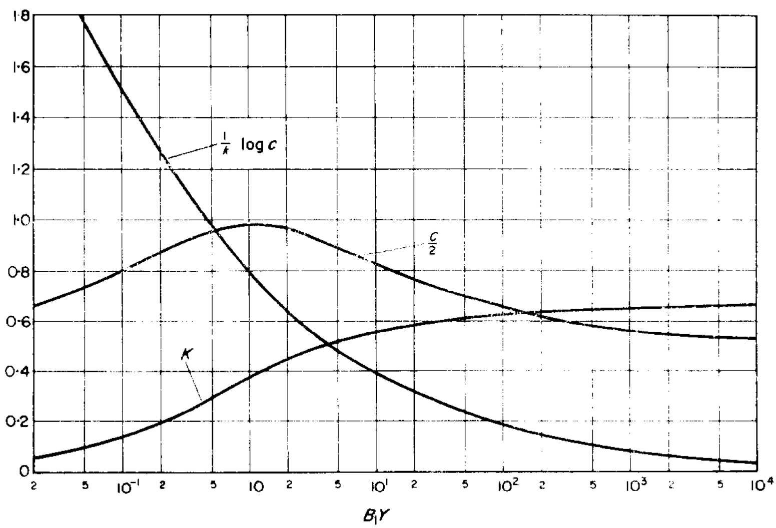

FIG. 11.

boiling it is necessary to examine the mechanism of the phenomenon.

Three topics are of interest in this case, namely (1) the volume history of the bubble $V(\tau)$, (2) the temperature distribution $\vartheta(y)$, and (3) the path of centre of gravity of the bubble $y(\tau)$.

The relation $\vartheta(D^+)$ was already found [equation (2.10)]. Substitution of equations (2.7), (2.8), (2.14) and (2.15) into (2.5) yields

$$\frac{\mathrm{d}D^+}{C'\,\mathrm{d}\,(y/l)} = \frac{\vartheta}{C_b\,\Delta t}\cdot\frac{D^{+s-1}}{1 - B_3\,D^{+2}}, \quad (6.1)$$

where

$$C' = 2\,C_\alpha\,C_w^{s-1}\,.\,C_\beta^{s-2}\,.\,C_b \cdot \frac{(Pr)^{1/3}\,(Re)}{(Nu)}\,.\,(Ar)^{s-1/2}. \quad (6.2)$$

Integrating equation (6.1) we obtain

$$C'\frac{y}{l} = \int_1^{D^+} (1 - B_3\,D^{+2})\,D^{+1-s}\,.\,\frac{C_b\,\Delta t}{\vartheta}\,\mathrm{d}D^+, \quad (6.3)$$

or $D^+ = D^+[C'(y/l)]$, which substituted into equation (2.10) yields

$$\frac{\vartheta}{C_b\,\Delta t} = f\left(C'\,\frac{y}{l}\right). \quad (6.4)$$

To find the relation $y(\tau)$ we use the equation (2.3); the result is

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = C_w\,.\,\frac{\nu'}{l}\,(Ar)^{1/2}\,.\,(1 - B_3\,D^{+2})$$

or

$$\frac{C'\,\mathrm{d}\,(y/l)}{\mathrm{d}\,(\nu'\tau/l^2)} = C'\,C_w\,(Ar)^{1/2}\,.\,(1 - B_3\,D^{+2}), \quad (6.5)$$

where the relation $D^+[C'(y/l)]$ was given above.

By integration we obtain

$$\frac{\nu'\tau}{l^2} = \frac{1}{C'\,C_w\,(Ar)^{1/2}}\int_0^{C'(y/l)} \frac{\mathrm{d}\,[C'\,(y/l)]}{1 - B_3\,D^{+2}}. \quad (6.6)$$

For $B_3 \ll 1$ (i.e. for $X \ll 1$) and small D^+ we obtain a reasonable linear dependence between y and τ, which was observed by Jakob ([24], p. 632).

To obtain the relation $\surd(\tau)$ we multiply equations (6.1) and (6.5); the result is

$$\frac{\mathrm{d}D^+}{\mathrm{d}\,(\nu'\tau/l^2)} = C'\,C_w\,(Ar)^{1/2}\,.\,\frac{\vartheta}{C_b\,\Delta t}\,.\,D^{+s-1}. \quad (6.7)$$

Introducing the reduced volume

$$V^+ = \frac{V}{V_0} = D^{+3} \quad (6.8)$$

we get

$$\frac{\mathrm{d}V^+}{\mathrm{d}\,(\nu'\tau/l^2)} = 3\,C'\,C_w\,(Ar)^{1/2}\,.\,\frac{\vartheta}{C_b\,\Delta t}\,.\,V^{+s+1/3} \quad (6.9)$$

and finally

$$\frac{\nu'\tau}{l^2} = \frac{1}{3\,C'\,C_w\,(Ar)^{1/2}}\cdot\int_1^{V^+} V^{+-(s+1/3)}\,.\,\frac{C_b\,\Delta t}{\vartheta}\,\mathrm{d}V^+, \quad (6.10)$$

where $\vartheta/(C_b\,\Delta t)$ is expressed in terms of V^+.

The integrals in equations (6.4), (6.6) and (6.10) should be evaluated numerically.

For $B_3 \approx 0$ and $B_1 > 10$ one can obtain considerably simpler relations. Namely, it follows from (2.10) that

$$\left(\frac{\vartheta}{C_b\,\Delta t}\right)^2 = 1 + \frac{1}{B_2'\,(C_b\,\Delta t)^2}\cdot\left[\frac{1}{1-s}(D^{+1-s} - 1) + \frac{B_1}{2+s}(D^{+-(2+s)} - 1)\right]$$

for $B_3 = 0$. Since for $B_1 > 10$ it is $B_1 \approx B_2$ we get

$$\left(\frac{\vartheta}{C_b\,\Delta t}\right)^2 = C_e^2 + (1 - C_e^2)\,.\left[D^{+-(2+s)} + \frac{2+s}{1-s}\cdot\frac{D^{+1-s} - 1}{B_2}\right].$$

If B_2 is sufficiently great and C_e sufficiently small, we obtain approximately (for smaller D^+)

$$\frac{\vartheta}{C_b\,\Delta t} \approx D^{+-(2+s/2)}$$

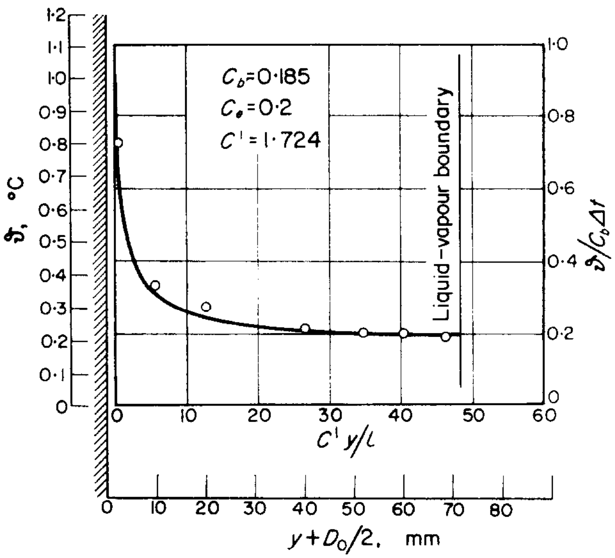

FIG. 12.

and

$$C'\frac{y}{l} = \int_1^{D^+} D^{+2-(s/2)}\,\mathrm{d}D^+ = \frac{1}{3-(s/2)} \cdot (D^{+3-(s/2)} - 1)$$

according to equation (6.3) for $B_3 = 0$. Thence

$$D^+ = \left[1 + \left(3 - \frac{s}{2}\right) C' \frac{y}{l}\right]^{1/[3-(s/2)]} \tag{6.11}$$

and

$$\frac{\vartheta}{C_b\,\Delta t} = \left[1 + \left(3 - \frac{s}{2}\right) . C' \frac{y}{l}\right]^{-(2+s/6-s)} \tag{6.12}$$

From equation (6.6) it follows for $B_3 = 0$

$$\frac{\nu'\tau}{l^2} = \frac{1}{C_w\,(Ar)^{1/2}} \cdot \frac{y}{l},$$

which substituted into equation (6.11) gives

$$D^+ = \left[1 + \left(3 - \frac{s}{2}\right) C'\,C_w\,(Ar)^{1/2} . \frac{\nu'\tau}{l^2}\right]^{1/[3-(s/2)]}$$

and

$$V^+ = \left[1 + \left(3 - \frac{s}{2}\right) C'\,C_w\,(Ar)^{1/2} . \frac{\nu'\tau}{l^2}\right]^{3/[3-(s/2)]} . \tag{6.13}$$

An example of relationships (6.4), (6.6) and (6.10) is shown in Figs. 12, 13 and 14 for an

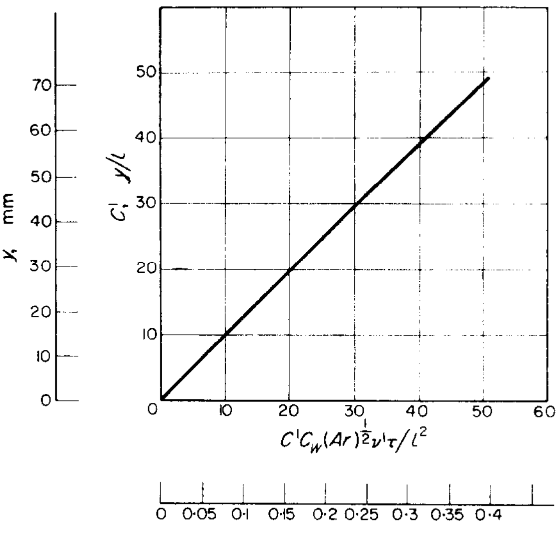

FIG. 13.

experiment of Jakob and Fritz [20] (water, p = 1·03 atm). The temperature distribution (Fig. 12) was also reproduced in [9] Fig. 11, p. 112). Values were quoted as follows: $q = 39\,300$ kcal/m²h and Δt = 10·6°C. Hence (Re) = 287·2 and (Nu) = 27·85. This point, marked in Fig. 6 by an arrow, lies to the left of the curve a so that assuming $\phi = 630$ (as for the curve a) we must assume an increased value of ψ. For

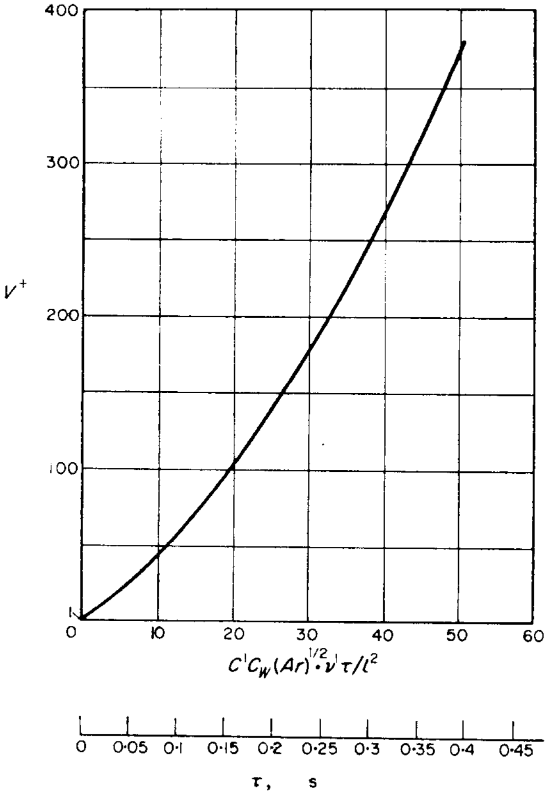

FIG. 14.

$\psi = 2{\cdot}83$ fair agreement with equation (2.12) is obtained. It follows that $Y = 19{\cdot}1$ ($B_2 = 365{\cdot}7$); $B_1 = 378{\cdot}0$ and $X = 3{\cdot}02 \times 10^{-5}$ ($B_3 = 1{\cdot}155 \times 10^{-3}$).

It was assumed tentatively $C_b = 0{\cdot}185$ and $C_e = 0{\cdot}2$; the corresponding temperature distribution, shown in Fig. 12, indicates that C_b might be assumed larger and C_e smaller. For the liquid–vapour boundary the following was obtained $C'(y/l) = 48{\cdot}28$. This corresponds to the value of $y = 70$ mm. Since $l = 2{\cdot}5$ mm it follows $C' = 1{\cdot}724$.

The value $C_b = 0{\cdot}185$ indicates a 81·5 per cent temperature drop in the boundary layer. Since the heat-transfer processes in the boundary layer are not stationary, and the variations in temperature distribution are much greater than in the bulk of liquid, the temperature drop could not be evaluated from this quasi-stationary theory. There is no guarantee that the obtained value $C_b = 0{\cdot}185$ is a universal constant.

From equation (6.2) the value of C_α was calculated. It is $C_\alpha = 15{\cdot}9$ for the analysed experiment. Substitution of ψ, C_b, C_e, C_β and C_α in equation (4.8) yields $C_t \approx 15{\cdot}0$.

Since all the constants were known, it was possible to mark the scales in Figs. 12, 13 and 14 in mm or s.

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APPENDIX A

Solution of equation (2.6) for $s = 0$ and $s = 1$

For $s = 0$ we obtain from equation (2.6)

$$B_1 = D^{+3} - B_2' \cdot \frac{D^{+3}}{1 - B_3 D^{+2}} \cdot \frac{d\vartheta^2}{dD^+} \tag{A.1}$$

or integrated with condition (2.9)

$$B_2' [C_b^2 (\Delta t)^2 - \vartheta^2] = 1 - D^+ - \frac{B_1}{2}\left(\frac{1}{D^{+2}} - 1\right) + B_3 \left[\frac{1}{3}(D^{+3} - 1) - B_1 \ln D^+\right] \tag{A.2}$$

Taking into account the condition (2.11) we get

$$B_2 = B_1 - 3 B_1^{1/3} + 2 - (2/3) B_3 [B_1 \ln B_1 + 1 - B_1]. \tag{A.3}$$

For $s = 1$ the equation (2.6) reads

$$B_1 = D^{+3} - B_2' \cdot \frac{D^{+4}}{1 - B_3 D^{+2}} \cdot \frac{d\vartheta^2}{dD^+}. \tag{A.4}$$

The equation, corresponding to (A.3), is

$$B_2 = B_1 - \ln B_1 - 1 - 3 B_3 \left[B_1 - \frac{3}{2} B_1^{2/3} + \frac{1}{2}\right]. \tag{A.5}$$

APPENDIX B

Mean temperature of the liquid at the heated surface

As it was indicated before, we operate not with the actual but with the mean temperature difference ϑ. If we extend the solution $\vartheta(y)$ into the boundary layer $(-(1/2)D_0 < y \leqslant 0)$ the result for $y = -(1/2)D_0$ will indicate approximately the mean temperature of the liquid at the surface. For this purpose we find $d\vartheta/dy$ for $y = 0$, combining equations (6.1) and (2.6); thus

$$\frac{d\vartheta}{dy} = C' C_b (1 - C_e^2) \cdot \frac{\Delta t}{l} \left(1 + \frac{s}{2}\right) \cdot \frac{D^{+3} - B_1}{B_2 D^{+4}}$$

and

$$\left(\frac{d\vartheta}{dy}\right)_{y=0} = - C' C_b (1 - C_e^2) \frac{\Delta t}{l} \cdot \left(1 + \frac{s}{2}\right) \cdot \frac{B_1 - 1}{B_2}. \tag{B.1}$$

Assuming linear temperature distribution in the boundary layer we obtain

$$\frac{\vartheta_m - C_b \Delta t}{D_0/2} = - \left(\frac{d\vartheta}{dy}\right)_{y=0}, \tag{B.2}$$

where

$$\vartheta_m = C_m \Delta t \tag{B.3}$$

is a hypothetic mean temperature difference of the liquid at the surface. Equating the left-hand sides of (B.1) and (B.2) and solving for C_m we get

$$C_m = C_b \left[1 + \frac{2 + s}{4} C' (1 - C_e^2) \cdot \frac{B_1 - 1}{B_2}\right]. \tag{B.4}$$

For the numerical example, discussed in Section 6, we obtain $C_m = 0{\cdot}392$ (for $s = 1/2$).

Now, after the departure of the bubble the colder liquid approaches the wall; the minimum temperature difference of this liquid may be estimated from Fig. 12 for $y = (1/2)D_0$. Its value is approximately $0{\cdot}6\ C_b \Delta t = 0{\cdot}11\ \Delta t$. The maximum temperature difference of the liquid at the wall is Δt. The arithmetic mean is therefore $0{\cdot}555\ \Delta t$. There is evidently some relation between this value and the value of $C_m \Delta t$.

Abstract—It is assumed in the following study that the heat flux in boiling heat transfer consists of three components: the first due to the flow of columns of bubbles; the second due to the molecular heat conduction in the liquid; and the third is due to the eddy convection. The latter is estimated from the data on the bubbling process. The differential equation obtained gives after integration a relationship between the local bubble diameter and the temperature difference. As a result it is found that $(Nu) \sim (Re)^{2/3}$ for sufficiently great Nusselt moduli under the assumption that the bubble population is inversely proportional to the radius of a nucleus. If one assumes that the number of bubbles per unit area is inversely proportional to the radius of nucleus to the m-th power, the relation

$$(Nu) \sim (Re)^{1+m/2+m}$$

may be obtained for large (Nu).

The theory is compared with experiments with satisfactory results. Furthermore the first crisis of boiling, the influence of the heated surfaces and the peculiarities of the nucleate boiling mechanism are analysed.

Résumé—On suppose dans l'étude suivante que le flux de chaleur dans le transport de chaleur par ébullition se compose de trois composantes: la première due à l'écoulement de chapelets de bulles; la seconde due à la conduction moléculaire de la chaleur dans le liquide; et la troisième est due à la convection turbulente. La dernière est estimée à partir des données sur la proccessus du bouillonnement. L'équation différentielle obtenue donne après intégration une relation entre le diamètre de bulle local et la différence de température. En résultat, on trouve que $Nu \sim (Re)^{2/3}$ pour des nombres de Nusselt suffisamment élevés avec l'hypothèse que le nombre de bulles est inversement proportionnel au rayon d'un germe. Si on suppose que le nombre de bulles par unité de surface est inversement proportionnel au rayon du germe à la puissance m, la relation:

$$(Nu) \sim (Re)^{1+m/2+m}$$

peut être obtenue pour de grands Nu.

La théorie s'accorde avec l'expérience d'une manière satisfaisante. De plus, on a analysé la première crise de l'ébullition, l'influence des surfaces chauffées et les particularités du mécanisme de l'ébullition par germes.

Zusammenfassung—In der folgenden Untersuchung des Wärmeüberganges beim Sieden wird angenommen, dass der Wärmefluss in drei Komponenten zerlegbar ist: erstens, in einen Fluss in den Blasensäulen; zweitens, in einen Fluss molekularer Wärmeleitung in der Flüssigkeit und drittens in einen Fluss durch Wirbelkonvektion. Letzter ist aus Werten der Blasenbildung abzuschätzen. Die Integration der Differentialgleichung führt zu einer Beziehung zwischen dem örtlichen Blasendurchmesser und der Temperaturdifferenz. Für genügend grosse Nusselt-Zahlen und mit der Annahme, dass die Blasendicke umgekehrt proportional dem Keimradius ist, folgt $Nu \sim Re^{2/3}$. Nimmt man an, dass die Zahl der Blasen pro Flächeneinheit umgekehrt proportional dem Keimradius zur m-ten Potenz ist, so ergibt sich für grosse Nu die Beziehung

$$Nu \sim Re^{1+m/2+m}$$

Die Theorie lässt sich mit Experimenten zufriedenstellend vergleichen. Darüberhinaus werden der erste Wendepunkt der Siedekurve, der Einfluss der Heizflächen und Besonderheiten des Mechanismus des Blasensiedens analysiert.